Relationships between the knowledge and beliefs about mathematics teaching and learning of two university lecturers in linear algebra

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In this study we explore the potential relationships between the specialised knowledge of two university lecturers in linear algebra and their beliefs about mathematics teaching and learning. We scrutinise the lecturers’ knowledge with the aid of the model Mathematics Teacher’s Specialised Knowledge (MTSK), from which the subdomains Knowledge of topics (KoT), Knowledge of mathematics teaching (KMT) and Knowledge of features of learning mathematics (KFLM) enable us to establish associations with their beliefs. We found that the beliefs manifested by these two lecturers about teaching methodology and subject significance are related to their knowledge of topics in terms of procedures and phenomenology, and also in terms of applications.

Keywords: specialised knowledge, beliefs, university lecturer, linear algebra.

INTRODUCTION

Amongst studies into mathematics education, teachers’ knowledge has received special attention. There has been found to be a connection between this knowledge and the teacher’s beliefs, and both contribute to the quality of teaching. The majority of studies into teachers’ knowledge and beliefs have explored changes to these two constructs, or they have considered them separately.

There have been few studies directed towards the relationship between university lecturers’ knowledge and their beliefs about teaching and learning mathematics. We presented the findings of one study, focusing on the specialised knowledge deployed by a lecturer in linear algebra when teaching the topic of matrices and determinants, in a paper at the CERME meeting in the Czech Republic (Vasco, Climent, Escudero-Ávila, & Flores-Medrano, 2015). The paper presented here develops this line of research, examining possible relationships between the specialised knowledge displayed by two linear algebra lecturers, as manifested in the examples they choose for teaching purposes, and their beliefs about teaching and learning mathematics.

MATHEMATIC TEACHERS’ SPECIALISED KNOWLEDGE

In the last few decades, teachers’ knowledge has attracted a great deal of interest from researchers, and various frameworks have been developed for the purpose. In our case, we have made use of the model Mathematics Teachers’ Specialised Knowledge (MTSK) (Carrillo, Climent, Contreras, & Muñoz-Catalán, 2013). This model was developed in response to the difficulties experienced in applying the model Mathematics Knowledge for Teaching (MKT) (Ball, Thames, & Phelps,
2008), specifically with respect to the characterisation and demarcation of the different subdomains into which it is constituted, and aims instead to represent the specialised nature of teachers’ knowledge as an integral and inseparable element of that knowledge.

The MTSK model comprises two knowledge domains: Mathematical Knowledge (MK) and Pedagogical Content Knowledge (PCK). MK is itself constituted by the subdomains Knowledge of topics (KoT), Knowledge of the structure of mathematics (KSM) (knowledge of the connections between different mathematical content) and Knowledge of practices in mathematics (KPM) (knowledge of the ways of proceeding characteristic of mathematical work). For its part, PCK includes the subdomains Knowledge of mathematics teaching (KMT), Knowledge of features of learning mathematics (KFLM) and Knowledge of mathematics learning standards (KMLS) (knowledge about the curriculum and the objectives and performance measures issued by professional and research associations). For details of the subdomains and categories making up the framework, readers are directed to previous CERME presentations (Carrillo et al., 2013; Vasco et al., 2015). In this paper we limit ourselves to a brief overview of the subdomains KoT, KMT and KFLM, as it is these which allowed us to establish the relationships between the teachers’ specialised knowledge and their beliefs about teaching and learning mathematics.

KoT is defined as a thorough, grounded knowledge of mathematical content. It comprises the categories of phenomenology and applications (phenomena associated with the meanings of a mathematical topic and ways a topic can be applied), properties and fundamentals (properties which fulfil a mathematical objective or are necessary to carry out a procedure), representations (the different ways a topic can be represented), definitions (descriptions and characterisations of a concept, including examples and related images), and procedures (How is something done? Under what conditions can something be done? Why is something done in this particular way? and What are the key features of the result?).

KMT, as the name suggests, concerns knowledge about the teaching of mathematics and includes the following categories: theories of teaching (that is, specific to mathematics education), material and virtual resources (books, whiteboards, software, and so on as tools for teaching mathematics), and activities, tasks and examples for teaching (examples and their potency for the topic in question).

KFLM is knowledge about how mathematics is learned. The chief focus is not on the student, but rather on mathematical content as the object of learning. The categories included here are: learning styles (theories of the cognitive development of the student), areas of strengths and weaknesses associated with learning (that is, of the student in regard to the content), the students’ forms of interacting with the content (student strategies), and the student’s motivation with regard to mathematics (students’ expectations about the content).
BELIEFS ABOUT TEACHING AND LEARNING MATHEMATICS

Teachers’ knowledge and beliefs interact (Charalambous, 2015), and these interactions can lead to a better understanding of both aspects (Flores and Carrillo, 2014). In a previous study (Turner, 2009), beliefs about mathematics and the teaching of mathematics were considered a component of knowledge of mathematical content and were incorporated into the Foundation dimension of the model Knowledge Quartet (Rowland, Huckstep, & Thwaites, 2005). In this paper we present possible connections between the subdomains of the specialised knowledge of two linear algebra teachers and their beliefs about the mathematics teaching and learning, and in this way advance our understanding of beliefs as part of the MTSK model.

We are aware that different positions can be taken in relation to the terms “beliefs” and “conceptions”. It is often the case that questions which might arise about beliefs can be framed in equivalent terms about conceptions. Indeed, given that the terms are used to mean different things in different studies, there is space for a significant degree of overlap between the two constructs. As the individual’s set of incontrovertible personal truths, beliefs are an important means of taking account of the affective aspects of the teacher’s personality. Conceptions, on the other hand, can be considered as a conceptual substrate, influencing all aspects of a cognitive nature and playing a key role in determining the teacher’s thinking and actions (Ponte, 1994, 1998). In this study we will use the term “beliefs” to refer indistinguishably to conceptions and beliefs (as discussed above), although aware that our focus is chiefly cognitive.

In order to study the lecturers’ mathematics teaching and learning beliefs (MTLB), we drew on the classifications in Carrillo & Contreras (1994) for tendencies (traditional, technological, spontaneous and investigative) and categories (methodology, subject significance, learning conception, student’s role, teacher’s role and assessment). Each category has corresponding indicators by which teachers’ beliefs can be inferred. The aim is to arrive at an interpretative description of the teacher’s beliefs, rather than make a definitive assignment to one tendency or another. What interests us is having available an instrument that enables us to identify beliefs and that these help us to understand the teacher’s knowledge, and vice versa.

METHODOLOGY

This is a qualitative study, taking a case-study design (Yin, 2003). It focuses on two lecturers teaching linear algebra in the first year of a degree course at the State Technical University of Quevedo, Ecuador, and who, for the purposes of this study, will be referred to as Jordy and Carlos. We attempt to provide an answer to the research question, ‘What interaction is there between the specialized knowledge of two linear algebra lecturers and their beliefs about mathematics teaching and
learning?’ In this paper, due to limitations of space, we will focus principally on the knowledge which is manifested in the use of examples.

The two lecturers were chosen for their willingness to take part in the study and our intrinsic interest in the teaching cohort to which they belonged, with a view to subsequent participation. Jordy is a graduate of the Educational Sciences faculty, specializing in mathematics, with 22 years’ experience in teaching mathematics at secondary level and 9 years at the university. Carlos is a geologist with 17 years’ experience of teaching mathematics in the university.

Data collection was carried out via class observations (using video recordings) and semi-structured interviews. The topic of matrices, determinants and systems of linear equation was chosen to be observed as it was first in the study programme for the course in linear algebra, and essential for tackling subsequent topics. The aim of this research project was to gain an understanding of the knowledge underlying the teaching of the two lecturers in linear algebra, and to scrutinise their beliefs about the teaching and learning of mathematics from the point of view of the relationships with their knowledge which we might uncover.

The analysis of these data followed the procedures of content analysis (Bardin, 1996), sifting through the teachers’ actions and statements for evidence of knowledge pertaining to the appropriate MTSK subdomains, and beliefs about mathematics teaching and learning, using throughout the categories outlined above – from the MTSK model in the case of knowledge, and the analytical tools in Carrillo & Contreras (1994) in the case of beliefs.

RESULTS

Tables 1, 2 and 3 summarise the relationships between the teachers’ knowledge and their beliefs. Both lecturers pursued procedural objectives with their classes, demonstrating an interest in their students gaining mastery of the operations and algorithms associated with the mathematical content in question. Jordy and Carlos ascribed an instrumental end to teaching matrices (in that they provide solutions to systems of linear equations or to find the value of a variable). This is made clear in their interviews:

Jordy: [In answer to what he wanted his students to learn about matrices] To be able to do the required operations, pose problems involving matrices and know how to solve them. Because matrices allow you to solve systems of equations.

Carlos: When we’re talking about matrices, we’re talking specifically about variables, and there are different procedures for finding variables when we apply matrices.

Evidence of knowledge about scenarios requiring the use of the content and potential applications (KoT – phenomenology and applications) is demonstrated in the case of
Jordy through mathematical situations (solving systems of equations, fundamentally), and in the case of Carlos through some situations which could arise in real life. The kind of knowledge displayed in these instances would seem to be consistent with their conceptions about the utility of the mathematics under consideration (that is, serving practical ends, with high value given to the ability to reproduce the content further on in time). Below are the relevant excerpts from their interviews:

Jordy: Now we need to turn our attention to vectors, and after that other areas based on matrices, so you see that this content is important because of what comes later in terms of calculus and differential equations.

Carlos: I give the students practical examples from real life, so that they can see that matrices are not just applicable to a specific science, but to any area, such as nutrition, sport, or a factory. In other words, matrices have multiples uses and applications.

There was also evidence of associations between the teachers’ knowledge about KoT – procedures and their conceptions about teaching methodology. Jordy chooses not to present the content as a unitary procedure; instead, he sets up a series of exercises which aim to reproduce the logical processes involved. Carlos, on the other hand, does decide to present the content in this way, and the lesson activity is marked by the repetition of exercises. In setting up the class in this way, Jordy manifests a depth of knowledge pertaining to the category KoT – procedures, which leads him to go beyond a plain exposition of the procedures in order to underline the reasons underlying their use. This knowledge might be associated with his knowledge regarding KoT – phenomenology and applications, and hence his content knowledge is located within mathematics. In the case of Carlos, the knowledge in evidence is of the type How is something done, and hence his knowledge would appear to be external to mathematics, attuned to an eminently utilitarian view of the subject.

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<thead>
<tr>
<th>Beliefs</th>
<th>MTSK</th>
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<tr>
<td>Methodology: replicating patterns of thought (Jordy) and mechanical (Carlos). Subject significance: Orientation (applicability), Objective (informative, utilitarian) (Jordy and Carlos)</td>
<td>KoT-procedures: How is something done? (Jordy and Carlos) and the reasoning underlying procedures (Jordy). KoT- phenomenology and applications: applications to mathematics itself (Jordy) and applications to real life (Carlos).</td>
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Table 1: Associations between beliefs and KoT of Jordy and Carlos

There are also associations between Jordy’s knowledge of examples (KoT – examples for teaching) and his conceptions about methodology. Jordy seems to conceive of the role of the lecturer as that of replicating the process of building
knowledge via a selection of examples which allow different features of the content to be highlighted. The extract below is from his introduction to the topic of multiplying matrices:

Jordy: In order to add matrices we need to meet a condition. What was it?
Student: They need to have the same dimensions.
Jordy: They need to have the same dimensions. In order to multiply matrices we also need to meet a condition. If we take matrix A […] What are the dimensions of this matrix?
Student: Two rows, three columns.
Jordy: The dimension is 2x3. To multiply two matrices we need the number of columns in the first matrix to be the same as the number of rows in the second matrix. If A looks like this, B has to have three rows, it doesn’t matter how many columns it has. Let’s imagine that matrix B is a column matrix […] It can be multiplied with this one, the only condition is that it has three rows. The dimensions of this matrix are . . .
Student: 3x1
Jordy: 3x1. If the matrix doesn’t meet this condition, then as you say it cannot be multiplied. Matrix B can also, for example, look like this […] [The lecturer writes two more matrices, 3x2 y 3x4.] What we do is we take the first row of matrix A and we multiply it by the items in the first column of matrix B […]

Jordy generally presented the content using three of four examples, in this case avoiding entering into the topic of multiplying square matrices of the same order so as to make it clear to the students how important it is to define the dimensions of the matrices to be multiplied in order to determine whether it can be carried out or not. In addition, he employed a wide variety of examples, which meant that he could focus students’ attention on the more salient features and on aspects providing a wide range of images. We associate this knowledge of examples with Knowledge of Mathematics Teaching (KMT).

Jordy’s conceptions are aligned with a kind of classroom practice favouring what could be called replicative structured exercises (that is, the student exercises are designed to encourage them to replicate the logical thought processes). He warns students about potential pitfalls along the way. This conception of practice is associated with his knowledge about errors and difficulties in learning (KFLM – areas of strengths and weaknesses associated with learning). Hence, when dealing with the topic of multiplying matrices he advises the students to define the dimensions in order to avoid falling into the common error of applying the algorithm for adding matrices, multiplying (in the case of matrices with the same dimensions) elements in corresponding positions (as he explains in an interview):
Jordy: The error that they can always fall into first is thinking that they have to multiply number by number according to the position it’s in. So, at least in this case, multiplying matrices, I tend to bang on about dimensions.

Thus, his knowledge of students’ learning difficulties with respect to the content (KFLM) is associated with his knowledge about examples for teaching content (KMT) and his conceptions about classroom practice (replicative structured exercises, with emphasis on warning students about pitfalls). At the same time, his knowledge of examples is related to his conception of the role of the teacher (presenting content by means of replicating the process of its construction).

Jordy displays knowledge of procedures (KoT) concerning to how it is done, under what conditions it can be done, and what the key features of the result are (Vasco et al., 2015). His Knowledge of Topics (KoT) would seem to underlie his analysis of difficulties (and possibly vice-versa) and is consistent with his concern to alert students to potential errors. Apparently, his knowledge and beliefs feed into each other, in that his beliefs can lead him to fix upon and explore something which supplies him with new knowledge, while the assimilation of this knowledge can attune his sensitivity towards the role of error.

<table>
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<tr>
<td>Methodology: replicating patterns of thought with emphasis on errors</td>
<td>KMT-examples for teaching: variability of examples</td>
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<tr>
<td>Teacher’s role: deliver content by replicating process of knowledge construction</td>
<td>KFLM-weaknesses associated with learning: transposition of algorithm for adding matrices to multiplication; failure to note dimensions of matrices</td>
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Table 2: Associations between Jordy’s beliefs and his KMT and KFLM

In Carlos’ case, his knowledge concerning the category of examples for teaching (KMT) is intertwined with his beliefs about subject significance, that is, that the point of the subject is essentially informative with practical applications in everyday life. To this end, the lecturer develops the work on matrices around a set of exercises intended to reproduce real life situations.

Carlos: I always give you practical examples so that you get an idea of how they can be applied […] You can see here that we’ve got two tables with information about different situations. The first gives different routes followed by four makes of car (3x4), while the second table shows the petrol consumption for each of the models over the five week days (4x5). If we put them into matrix form will they be the same or not?

Student: No, they won’t.

Carlos: No, but that isn’t what matters in this case. Here, the layout they’re in is the right one to multiply them because what we do is multiply the rows by the
columns; so the number of columns in the first matrix coincides with the number of rows in the other one […] We have to multiply the element which is in the first row, first column of table T by the element which is in the first row, first column of table G […].

By going through exercises of this kind, the need to define the operation (the multiplication of matrices) emerges, and this approach is consistent with the lecturer’s beliefs regarding framing what needs to be learned in practical terms. He is concerned that the students should learn the procedure, and goes to some lengths to demonstrate the application of the content to real life situations. In this case, in terms of the lecturer’s beliefs, class activity would seem to be based on the repetition of such exercises, with less emphasis on the logical processes involved and on errors (unlike the case of Jordy). The lecturer’s KoT is evident in procedures (how it is done, essentially) and phenomenology (application of content to other fields). On the other hand, regarding his pedagogical content knowledge, KMT is evident in his choice of examples, associated with his knowledge about how the content can be applied (KoT). The knowledge that Carlos manifests seems to be chiefly related to the practical purposes by which he views the contents, and his emphasis on procedural performance.

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<th>Beliefs</th>
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<tr>
<td>Methodology: mechanical repetition of exercises</td>
<td>KMT-examples for teaching: examples of applications to real life</td>
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<tr>
<td>Subject significance: purpose (informative and practical)</td>
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Table 3: Associations between Carlos’ beliefs and his KMT

CONCLUSIONS AND IMPLICATIONS FOR FUTURE RESEARCH

Having established the relationships between them, we can see how the knowledge and beliefs of the two lecturers studied interact. Their conceptions enable us to better understand their knowledge and vice-versa. Their conceptions about the purpose of the content and the objectives they prioritise are consistent with the KoT (regarding procedures and applications) which each of them manifests in their practice. Their conceptions about class practice activities (exercises which replicate patterns of thought versus activities focusing on mechanical repetition, with possible emphasis on errors), and about the role of the lecturer indicate an association with the KMT that each demonstrates (in terms of knowledge of examples); in the case of emphasising error avoidance, there is an association with their KFLM (in terms of knowledge of errors). Both aspects (beliefs and knowledge) are reflected in the elements they give emphasis to in the course of their teaching. Hence, dealing with the multiplication of matrices, one of the lecturers chooses to emphasise the conditions required to apply
the procedure and the avoidance of potential errors, the other the applications to real-life situations.

Teachers’ knowledge and ways of thinking are essential to the teaching of mathematics, and need to be understood in order for teachers to be helped to improve their practice, and consequently their students’ learning (Chapman, 2015). We have carried out an initial approach towards establishing relationships between teachers’ knowledge and their conceptions about the teaching and learning of mathematics, and this contributes to our understanding of how and why teachers do what they do in the classroom.

Detailed analysis of teachers’ knowledge and conceptions about mathematics teaching and learning can help us gain a broader vision of the relationships between these two constructs. Furthermore, it would be interesting to explore further where the associations that have been uncovered lead, how they might influence, for example, the teacher’s beliefs in generating specialized knowledge.

REFERENCES


